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Conjugate heat transfer of a plate fin in a second-grade fluid flow

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Abstract—A conjugate heat transfer problem of a second-grade viscoelastic fluid past a plate fin was studied. Governing equations, including heat conduction equation of a fin, and continuity equation, momentum equation and energy equation of a second-grade fluid, were analyzed by a combination of a series expansion method, the similarity transformation and a second-order accurate finite-difference method. Solutions of a stagnation flow ($\beta = 1.0$) at the fin tip and a flat-plate flow ($\beta = 0$) on the fin surface were obtained by a generalized Falkner–Skan flow derivation. These solutions were used to iterate with the heat conduction equation of the fin to obtain distributions of the local convective heat transfer coefficient and the fin temperature. Ranges of dimensionless parameters, the Prandtl number (Pr), the elastic number (E) and the conduction–convection coefficient (N_{cc}) are from 0.1 to 100, 0.001 to 0.3, and 0.5 to 2.0, respectively. Results indicated that elastic effect in the flow can increase the local heat transfer coefficient and enhance the heat transfer of a fin. Also, same as results from Newtonian fluid flow and conduction analysis of a fin, a better heat transfer is obtained with a larger N_{cc} and Pr. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

It is standard practice in the conventional heat conduction analysis to assume that the local forced convective heat transfer coefficient on the surfaces of a fin is a constant. However, evidences in the literature have demonstrated that the local convective heat transfer coefficient can experience substantial variations along the surfaces of a fin. These variations may be caused by non-uniformities in both velocity and temperature distributions in the flow adjacent to the surfaces of a fin. Basically, the convective heat transfer coefficient on the surfaces of a fin is established by a highly coupled interaction between thermal states of the fin and the surrounding flow. The forced-driven and temperature-dependent nature of the interaction on interfaces cause the flow and the temperature fields to be specific according to the temperature distribution along the surfaces of a fin.

Since the flow and temperature fields have a strong influence on the convective heat transfer coefficient that, in turn, strongly affects the fin temperature distribution, the tightness of the coupling is apparent. Any first-principle analysis of a fin must deal with the energy conservation equation of the fin, the equations of mass, momentum and energy conservation in the surrounding fluid. Even in the coupling, apparent variations in the local thermophysical properties and the fluid flow temperature distributions can still be calculated by analyzing the coupled equations of the flow and the fin. This means that the flow and temperature fields in the fluid and the temperature distribution along the surfaces of a fin must be solved simultaneously in a heat transfer problem of a fin/fluid system. Energy equations of the fin and the flow are coupled by conditions of a temperature continuity and a heat flux continuity at the solid–fluid interface in the analysis of this conjugate problem. It is worth mentioning that these continuity conditions make the present conjugate problem solvable. A special feature of the conjugate analysis is that the magnitude and distribution of the convective heat transfer coefficient on the surfaces of a fin are not prescribed in advance, but are the outcome of the solution.

In order to include non-Newtonian fluids in conventional studies of conjugate problems, conjugate heat transfer analysis of a fin in a second-grade viscoelastic fluid flow is the major concern of the present investigation. The problem considered a fin that transfers heat to or from a surrounding second-grade fluid flow by the forced convection. A friction–reduction phenomenon of some dilute polymer solutions or polymer fluids (some of these fluids, which can be formulated by the model used in the present study, are termed second-grade fluids) is a well-known fact in the studies of non-Newtonian fluid flows [1]. Thus, if we use a non-Newtonian fluid as the coolant of the cooling systems or heat exchangers might greatly reduce the required pumping power. Therefore, fundamental analysis of the flow field of non-Newtonian fluids in a boundary layer adjacent to a fin or an extended surface is very important, and is also an

NOMENCLATURE

$\mathbf{A}_1$, $\mathbf{A}_2$	kinematic tensors
$\mathbf{b}$	body force [N m^{-3}]
E	elastic parameter
f_0	zero-order dimensionless velocity function
f_1	first-order dimensionless velocity function
g_0	zero-order dimensionless temperature function
g_1	first-order dimensionless temperature function
h	local heat transfer coefficient [W m^{-2} K^{-1}]
$\hat{h}$	dimensionless local heat transfer coefficient
k	thermal conductivity of the fluid [W m^{-1} K^{-1}]
k_f	fin thermal conductivity of the fin [W m^{-1} K^{-1}]
m	wedge angle index
L	characteristic length or fin length [m]
N_{cc}	conduction–convection parameter
Nu	Nusselt number
P, P^*	pressures [N m^{-2}]
Pr	Prandtl number
q	local heat transfer rate of the fin [W]
Re	Reynolds number
Re_L	Reynolds number, $u_\infty L/\nu$
Re_x	Reynolds number; $u_\infty x/\nu$
$\mathbf{T}$	stress tensor
T_e	flow temperature at the outer edge of the boundary layer [K]
T_f	fin temperature at x [K]
T_0	fin base temperature [K]
T_∞	fluid temperature [K]
TT	constant related to the wedge angle and temperature gradient at the wall
$TT0$	zero-order part of TT
$TT1$	first-order part of TT
t	fin half thickness [m]
u_e	edge velocity [m s^{-1}]
U^*	characteristic velocity
u, v	horizontal and vertical flow velocities [m s^{-1}]
U, V	dimensionless horizontal and vertical flow velocities
$\mathbf{V}$	velocity vector
X	dimensionless coordinate (x/L)
x, y	horizontal and vertical coordinates [m]
X, Y	dimensionless horizontal and vertical coordinates.

Greek symbols

$\alpha_{1,2}$	first and second normal stress coefficients
β	shape factor
η	dimensionless similarity variable
θ_0	dimensionless temperature $(T-T_e)/(T_f-T_e)$
θ_f	dimensionless fin temperature $(T_f-T_e)/(T_0-T_e)$
μ	dynamic viscosity [kg s^{-1} m^{-1}]
ν	kinematic viscocity [m^2 s^{-1}]
ξ	dimensionless local parameter
ρ	density of the fluid [kg m^{-3}]
Φ	potential function
ψ	stream function.

essential part in the area of the fluid dynamics and heat transfer. Understanding boundary layer flows and heat transfer of non-Newtonian fluids has become important in recent years. Srivatsava [2], and Rajeswari and Rathna [3] studied the non-Newtonian fluid flow near a stagnation point. Mishra and Panda [4] analyzed the behavior of second-grade viscoelastic fluids under the influence of a side-wall injection in an entrance region of a pipe flow. Rajagopal *et al.* [5] studied a Falkner–Skan flow field of a second-grade viscoelastic fluid. Massoudi and Ramezan [6] studied a wedge flow with suction and injection along walls of a wedge by the similarity method and finite-difference calculations. Hsu *et al.* [7] also studied the flow and heat transfer phenomena of an incompressible second-grade viscoelastic fluid past a wedge with suction or injection. An excellent review of boundary layers in nonlinear fluids was recently written by Rajagopal [8]. These are related studies to the present investigation about second-grade fluids. The viscoelastic nature of a second-grade fluid is found in some dilute polymer solutions or in polymer fluids. These fluids exhibit both the viscous and elastic characteristics. The same as Newtonian fluids, the viscous property is due to the transport phenomenon of the fluid molecules. The elastic property is due to the chemical structure and configuration of the polymer molecule. The term 'elastic' means that the viscoelastic fluid 'remembers' where it was. Macromolecules act as small rubber bands and tend to snap back when the external forces are removed and, hence, produce 'elastic recoil' of the fluid. Detailed information of viscoelastic fluid can be found in books of rheology.

The system to be analyzed in the present study is a flat plate fin submerged in a second-grade viscoelastic fluid flow. Due to the coupling nature between the fin and the fluid, the present analysis is different from previous researches concerning forced convection about a flat-plate fin. Those studies have dealt primarily with a plate having prescribed convective heat transfer coefficient that yield similar or non-similar solutions [9, 10]. There are some related conjugate problems concerning a fin in a Newtonian flow, for instance, a complete model study about the forced convection on a rectangular fin has been investigated by Sparrow and Chyu [11]; the effect of the Prandtl number on the heat transfer from a rectangular fin has been studied by Sunden [12]. Also, Luikov and his co-workers solved the conjugate forced convective problem along a flat-plate both numerically [13] and analytically [14–16]. The analysis of the conjugate heat transfer problem encompasses simultaneous solutions for the heat conduction equation for the fin and the boundary layer equations for the adjacent fluid. These solutions are governed by two dimensionless parameters, one of which is termed the conduction–convection number (N_{cc}) and the other, the Prandtl number (Pr). Values of the conduction–convection number are selected, in general, to cover the entire range of feasible operating conditions and the Prandtl number varies from 0.1 to 100.

The objective of the present analysis is to study the conjugate heat transfer of a plate fin cooled or heated by a high or low Prandtl number, second-grade viscoelastic fluid with various conduction–convection parameters. An extension of previous works is then performed to investigate the conjugate heat transfer of a second-grade viscoelastic fluid past a plate fin. A schematic diagram of the flat-plate fin is shown in Fig. 1 to illustrate the physical situation and symbols of parameters needed for the analysis. Two types of flow fields, a stagnation flow (flow at the fin tip) and a flat-plate flow (flow on side surfaces of the fin), respectively, are included. The Rivlin–Ericksen model for grade-two fluids is used in the momentum equations. The effects of dimensionless parameters, the Prandtl number (Pr), the elastic number (E) and the conduction–convection coefficient (N_{cc}) are main interests of the study. Flow and temperature fields of the stagnation flow and the flat-plate flow are analyzed by utilizing the boundary layer concept to obtain a set of coupled momentum equations and energy equations. A similarity transformation with wedge-type parameters and a series expansion method are then used to convert the nonlinear, coupled partial differential equations to a set of nonlinear, decoupled ordinary differential equations. In the present conjugate problem, these decoupled equations and the conduction equation of the fin are then solved iteratively to obtain the temperature distribution and the local convective heat transfer coefficient along the fin by a second-order accurate finite difference method. While the difference form of the fin conduction equation has previously been solved by either a relaxation procedure [17, 18] or a direct matrix-inverse method [19, 20] and the Runge–Kutta integration method [21, 22],

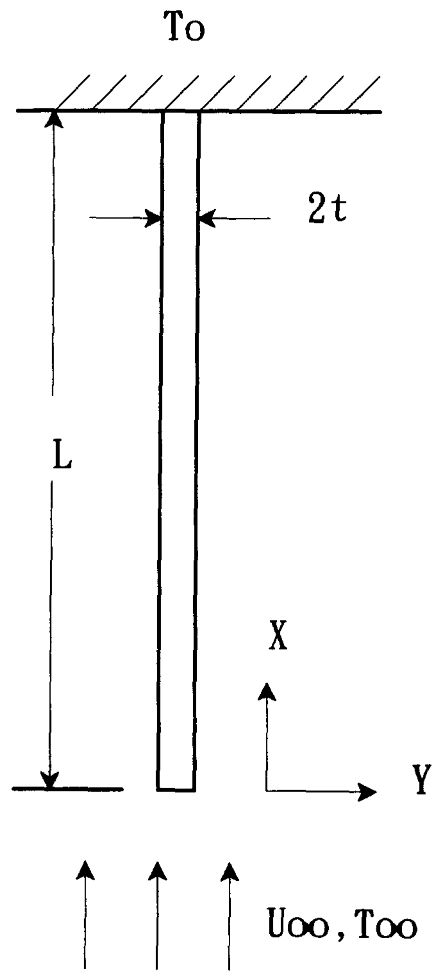

Fig. 1. Physical model and coordinate system.

a simple and stable direct Gauss elimination method [23] is used in the present study.

2. THEORY AND ANALYSIS

The Rivlin–Ericksen model [24] for a homogeneous, non-Newtonian, second-grade viscoelastic fluid is used in the present wedge flow. The model equation is expressed as follows:

$$\mathbf{T} = -P\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \tag{1}$$

where P is pressure, μ is dynamic viscosity, α_1 and α_2 are first and second normal stress coefficients which are related to the material modulus, and represent the elastic characteristic of the fluid. The kinematic tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined as

$$\mathbf{A}_1 = \operatorname{grad}\mathbf{V} + (\operatorname{grad}\mathbf{V})^{\mathrm{T}} \tag{2}$$

$$\mathbf{A}_2 = \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A}_1 + \mathbf{A}_1 \cdot (\operatorname{grad}\mathbf{V}) + (\operatorname{grad}\mathbf{V})^{\mathrm{T}} \cdot \mathbf{A}_1 \tag{3}$$

where $\mathbf{V}$ are velocities and $\mathrm{d}/\mathrm{d}t$ is the material time derivative. As done by Rajagopal [25], the present researchers substituted eqn (1) into momentum equations

$$\rho \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{V} = \operatorname{div} \mathbf{T} + \rho \mathbf{b} \quad (4)$$

and assumed that the fluid is incompressible and the flow is in isochoric motion to obtain

$$\operatorname{div} \mathbf{V} = 0. \quad (5)$$

For the steady, two-dimensional laminar flow under conservative body force $\mathbf{b}$, the following were defined:

$$P^* = P - \left(2\alpha_1 + \frac{\alpha_2}{2}\right)\left(\frac{\partial u}{\partial y}\right)^2 + \rho\Phi \quad (6)$$

$$\mathbf{b} = \nabla\Phi. \quad (7)$$

From Bernoulli's principle and the substitution of the edge velocity u_e, the following was obtained:

$$u_e \frac{\partial u_e}{\partial x} = -\frac{1}{\rho}\frac{\partial P^*}{\partial x}. \quad (8)$$

Consequently, one can eliminate the pressure term in the momentum equation and obtain the dimensionless boundary-layer equations

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (9)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = U_e \frac{\mathrm{d}U_e}{\mathrm{d}X} + \frac{\partial^2 U}{\partial Y^2} + E\left[\frac{\partial}{\partial X}\left(U\frac{\partial^2 U}{\partial Y^2}\right) + \frac{\partial U}{\partial Y}\frac{\partial^2 V}{\partial Y^2} + V\frac{\partial^3 U}{\partial Y^3}\right] \quad (10)$$

where $E = \alpha_1 Re_L/\rho L^2$, Re_L is the Reynolds number and L is the characteristic length. The corresponding dimensionless parameters are

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}\sqrt{Re_L}, \quad U = \frac{u}{U^*},$$
$$V = \frac{v}{U^*}\sqrt{Re_L}, \quad U_e = \frac{u_e}{U^*}. \quad (11)$$

The dimensionless boundary conditions are

$$\begin{aligned} Y = 0 &\quad U = 0 \\ &\quad V = 0 \\ Y \to \infty &\quad U \to U_e(X). \end{aligned} \quad (12)$$

By using the stream function ψ one can define

$$U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X} \quad (13)$$

and substitute into eqn (10) to get

$$\frac{\partial \psi}{\partial Y}\frac{\partial^2 \psi}{\partial X \partial Y} - \frac{\partial \psi}{\partial X}\frac{\partial^2 \psi}{\partial Y^2} = U_e \frac{\mathrm{d}U_e}{\mathrm{d}X} + \frac{\partial^3 \psi}{\partial Y^3} + E\left[\frac{\partial}{\partial X}\left(\frac{\partial \psi}{\partial Y}\frac{\partial^3 \psi}{\partial Y^3}\right) - \frac{\partial^2 \psi}{\partial Y^2}\frac{\partial^3 \psi}{\partial Y^2 \partial X} - \frac{\partial \psi}{\partial X}\frac{\partial^4 \psi}{\partial Y^4}\right]. \quad (14)$$

The boundary conditions are written as

$$\begin{aligned} Y = 0 &\quad \frac{\partial \psi}{\partial Y} = 0, \quad \frac{\partial \psi}{\partial X} = 0 \\ Y \to \infty &\quad \frac{\partial \psi}{\partial Y} \to U_e(X). \end{aligned} \quad (15)$$

The viscoelastic model is applicable for diluted polymer fluids under retarded-motion expansion. So one can assume $E \ll 1$ and expand the stream function ψ with respect to E as

$$\psi = \psi_0(X, Y) + E\psi_1(X, Y) + \cdots + E^n\psi_n(X, Y). \quad (16)$$

Substituting eqn (16) into eqns (14) and (15), and introducing the similar transformation parameters

$$\eta = \left(\frac{m+1}{2}\right)^{1/2} X^{(m-1)/2} Y \quad (17)$$

$$f_0(\eta) = \left(\frac{m+1}{2}\right)^{1/2} \psi_0 X^{-(m+1)/2} \quad (18)$$

one can obtain a set of nonlinear ordinary differential equations from the concepts of perturbation technique and power series expansion. The equation of the zeroth-order term, f_0, is of the form

$$f_0''' + f_0 f_0'' + \beta[1 - (f_0')^2] = 0 \quad (19)$$

where $\beta = 2m/(m+1)$ is the shape factor of the wedge. Also from the potential flow theory, the edge velocity U_e is expressed as

$$U_e = X^m. \quad (20)$$

The boundary conditions are then written as

$$\begin{aligned} f_0(0) &= 0 \\ f_0'(0) &= 0 \\ f_0'(\infty) &\to 1. \end{aligned} \quad (21)$$

Similarly, by assuming

$$f_1(\eta) = \left(\frac{2}{m+1}\right)^{1/2} \psi_1 X^{(1-3m)/2} \quad (22)$$

and performing the similarity transformation, one can also obtain a nonlinear ordinary differential equation

$$\frac{m+1}{2}f_1''' + \frac{m+1}{2}f_0 f_1'' - (3m-1)f_0' f_1' + \frac{3m-1}{2}f_0'' f_1 + (3m-1)f_0' f_0''' - \frac{3m-1}{2}(f_0'')^2 - \frac{m+1}{2}f_0 f_0'''' = 0 \quad (23)$$

for the first-order term, f_1. The corresponding boundary conditions of the equation are

$$f_1(0) = f_1'(0) = 0, \quad f_1'(\eta \to \infty) \to 0. \quad (24)$$

In the present study, we simply set $\beta = 0$ ($m = 0$) and $\beta = 1.0$ ($m = 1.0$), respectively, to represent a flat-plate flow and a stagnation flow. Consequently, the velocity distribution can be obtained by solving eqns (19)–(21) and (23)–(24) with numerical methods. (In conjugate analysis, these equations are solved with energy equations of the fluid and the fin; detail steps are described in the latter part of this section.)

By introducing the non-dimensional temperature:

$$\theta = \frac{T - T_e}{T_f - T_e} \quad (25)$$

the non-dimensional energy equation (neglecting the viscous dissipation) in the boundary layer is written as

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \quad (26)$$

with the boundary conditions

$$\theta(X, 0) = 1, \quad \theta(X, \infty) = 0. \quad (27)$$

To utilize the concept of the local similarity transformation, one can define

$$\xi = X^{m-1} \quad (28)$$

and assume that the non-dimensional energy equation can be expanded according to E as

$$\theta = g_0(\xi, \eta) + E g_1(\xi, \eta) + \cdots + E^n g_n(\xi, \eta). \quad (29)$$

Finally, by substituting eqn (29) into eqn (26), the zero-order equation with respect to E and corresponding boundary conditions are

$$\frac{\partial^2 g_0}{\partial \eta^2} + Pr \cdot f_0 \frac{\partial g_0}{\partial \eta} = Pr \frac{2(m-1)}{m+1} \cdot \xi \cdot f_0' \cdot \frac{\partial g_0}{\partial \xi} \quad (30)$$

$$g_0(\xi, \eta = 0) = 1 \quad g_0(\xi, \eta \to \infty) = 0. \quad (31)$$

The first-order equation and boundary conditions are

$$\frac{\partial^2 g_1}{\partial \eta^2} + Pr\left(f_0 \frac{\partial g_1}{\partial \eta} + \frac{3m-1}{2} \cdot \xi \cdot f_1 \cdot \frac{\partial g_0}{\partial \eta}\right) = Pr\left(\frac{2(m-1)}{m+1} \cdot f_0' \cdot \xi \cdot \frac{\partial g_1}{\partial \xi} + (m-1) \cdot f_1' \cdot \xi^2 \cdot \frac{\partial g_0}{\partial \xi}\right) \quad (32)$$

$$g_1(\xi, \eta = 0) = 0, \quad g_1(\xi, \eta \to \infty) = 0. \quad (33)$$

One can solve eqns (30)–(33) by neglecting terms with ξ-derivatives (a local similar concept) to obtain temperature distributions. The heat flux on the surface of the fin is

$$q_w = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} = h(T_f - T_e) \quad (34)$$

and with some manipulations the local Nusselt number can be expressed as

$$Nu_x = hx/k = -\left(\frac{m+1}{2}\right)^{1/2} Re_x^{1/2} \left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0}. \quad (35)$$

The corresponding local heat transfer convective heat coefficient can be written as

$$h = -(k/x)\left(\frac{m+1}{2}\right)^{1/2} Re_x^{1/2} \left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0}. \quad (36)$$

The constant, related to the wedge angle and the temperature gradient in eqns (35) and (36), may be expressed as

$$TT = -\left(\frac{m+1}{2}\right)^{1/2} \left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0} \quad (37)$$

or expanded according to the order of E as

$$TT0 = -\left(\frac{m+1}{2}\right)^{1/2} \left.\frac{\partial g_0}{\partial \eta}\right|_{\eta=0} \quad (38)$$

and

$$TT1 = -\left(\frac{m+1}{2}\right)^{1/2} \left.\frac{\partial g_1}{\partial \eta}\right|_{\eta=0}. \quad (39)$$

The formulation of the first analysis principle for forced convection along a fin involves the energy conservation for the fin and the boundary layer equations for the flow. For a slender fin, ample evidence based on finite difference solutions shows that a one-dimensional model is adequate [26]. The fin temperature at any x location serves as the wall temperature for the adjacent fluid and is denoted as $T_f(x)$. The energy equation for the fin may be written in two different forms, depending on how the coupled-fin/boundary-layer problem is solved. The method used here involves a succession of consecutive iteration solutions for the fin and the boundary-layer flow, with the sequence continued until there is no change (within a preset tolerance) between the nth iteration and the $(n-1)$ iteration. Within each iteration information must be transferred from the boundary-layer solution which is current for that period and be used as input to update the fin solution. This information may be either in the form of the local heat flux $g(x)$ or the local forced convective heat transfer coefficient $h(x)$. Both $q(x)$ and $h(x)$ are available from the current wedge-type boundary layer solution. The fin energy equation can be expressed as

$$d^2T_f/dx^2 = q/k_f t \quad (40)$$

or

$$d^2T_f/dx^2 = (h/k_f t)(T_f - T_e) \quad (41)$$

where k_f and t are the thermal conductivity and the half thickness of the fin, respectively. For the solutions of either eqns (40) or (41) at a given cycle of the iterative procedures, h and q can be regarded as known quantities.

At first glance, it appears advantageous to use eqn (40) rather than eqn (41) because it is easier to solve; however, eqn (41) is employed in the solution scheme. The choice made was based on experience, which has shown that at any stage of an iterative cycle h is closer to the final converged result than q. Thus, eqn (41) is chosen to obtain rapid convergence of the iterative procedure, whereby this objective is satisfactorily fulfilled, as will be documented shortly. Equation (41) is recast in a dimensionless form by the substitutions

$$X = x/L, \quad \theta_f = (T_f - T_e)/(T_0 - T_e) \quad (42)$$

where T_0 is the base temperature of the fin, so that

$$d^2\theta_f/dX^2 = \hat{h}N_{cc}\theta_f \quad (43)$$

with boundary conditions

$$\theta_f = 1 \quad (X = 1), \quad k_f\frac{d\theta_f}{dX} + h\theta_f = 0 \quad (X = 0). \quad (44)$$

The exact solution of the above differential equation and boundary conditions (a conventional solution with constant h and used only for comparison) is

$$\theta_f = \frac{\cosh\sqrt{\hat{h}N_{cc}}x - (h/\sqrt{\hat{h}N_{cc}}k)\sinh\sqrt{\hat{h}N_{cc}}x}{\cosh\sqrt{\hat{h}N_{cc}}L - (h/\sqrt{\hat{h}N_{cc}}k)\sinh\sqrt{\hat{h}N_{cc}}L} \quad (45)$$

where N_{cc} is the conduction–convection number and is defined as

$$N_{cc} = (kL/k_f)Re_L^{1/2}. \quad (46)$$

The quantity $\hat{h}$ is a dimensionless form of the local convective heat transfer coefficient and can be written as

$$\hat{h} = (hL/k)Re_L^{-1/2}. \quad (47)$$

By the way, the Biot number is not an appropriate parameter in the present problem because the heat transfer coefficient varies with x and is also unknown prior at the beginning of the computations.

These conjugate ordinary differential equations are discretized by a second-order accurate central difference method, and a computer program has been developed to solve these equations. To avoid errors in discretization and calculation processing and to ensure the convergence of numerical solutions, some conventional numerical procedures have been applied in order to choose a suitable grid size ($\Delta\eta = 0.05$–0.1), a suitable η range and ξ or X positions, etc., and a direct gauss elimination method with Newton's method [27] is used in the computer program to obtain solutions of these difference equations. Calculation steps of the entire conjugate system are as follows:

(1) Estimate the fin temperature distribution $T_f(x)$.
(2) Solve flow fields [eqns (19), (21), (23)–(24) and (30)–(33)] and the local convective heat-transfer coefficient [eqn (47)] according to the local Prandtl number, elastic parameter, and the local fin temperature for both the stagnation flow ($\beta = 1.0$) and the flat-plate flow ($\beta = 0$) from the related equations.
(3) Solve the heat-conduction equation of the fin [eqn (43)] with the renewed local convective heat-transfer coefficient.
(4) Compute thermodynamic fluid properties from the fin temperature and free-stream temperature.

The sequence 2 to 4 is repeated until an acceptable convergence for fin temperature has been reached. The conditions of continuity in the heat flux and temperature at the fluid–solid interface are then satisfied and all relevant heat transfer characteristics can be calculated.

3. RESULTS AND DISCUSSION

Comparisons between the exact solution and numerical solution obtained by the present numerical method for heat conduction in the fin with the convective heat transfer coefficient, $h = 25$ and 50, are shown in Figs 2 and 3, respectively. Results exhibited favorable agreement between both solutions, and also indicated that the finite difference method used in the present study is adequate.

Many previous studies of conventional and conjugate problems did not consider convective effects of the stagnation flow at the fin tip, but simply substituted the convective condition by an adiabatic boundary condition. However, from the fin-flow configuration shown in Fig. 1, the heat transfer at the tip of the fin should not be ignored. It is important to include stagnation flow effects at the tip point of the fin in either conventional heat transfer problems or conjugate problems.

A generalized Falkner–Skan flow derivation is used to analyze a stagnation flow (shape factor $\beta = 1.0$) at the fin tip and a flat-plate flow ($\beta = 0$) on fin surfaces. A second-order accurate finite difference method is used to obtain solutions of these equations. Comparing $f_0''(0)$ and $f_1''(0)$ to results of [5] at various values of β showed a good agreement and these values are listed in Table 1. Also, computed values of $TT0$ at various values of Pr for flat-plate flow are consistent with Ref. [28], and are listed in Table 2. These tables indicated that the present results are correct, and the numerical method used is adequate.

For the stagnation flow, self-similar solutions for

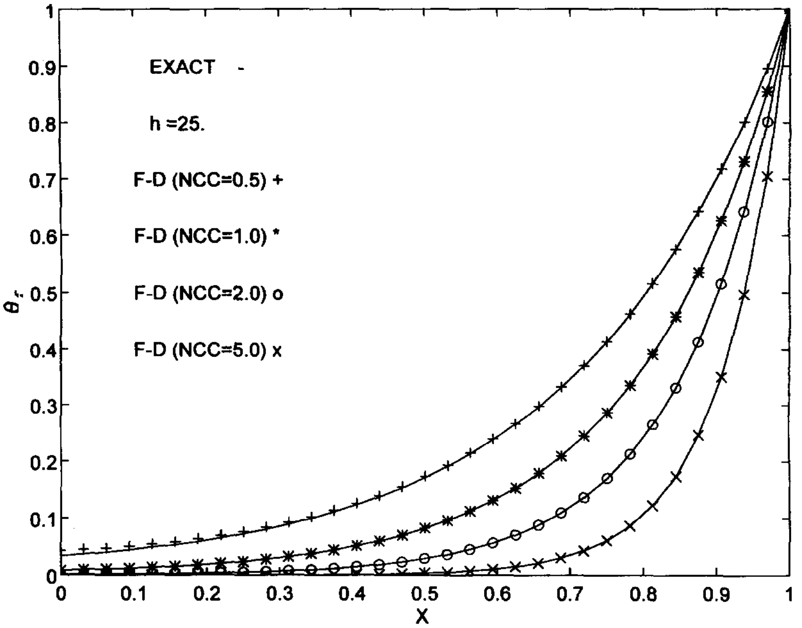

Fig. 2. Fin temperature distributions computed from FDE and exact solution for $h = 25.0$.

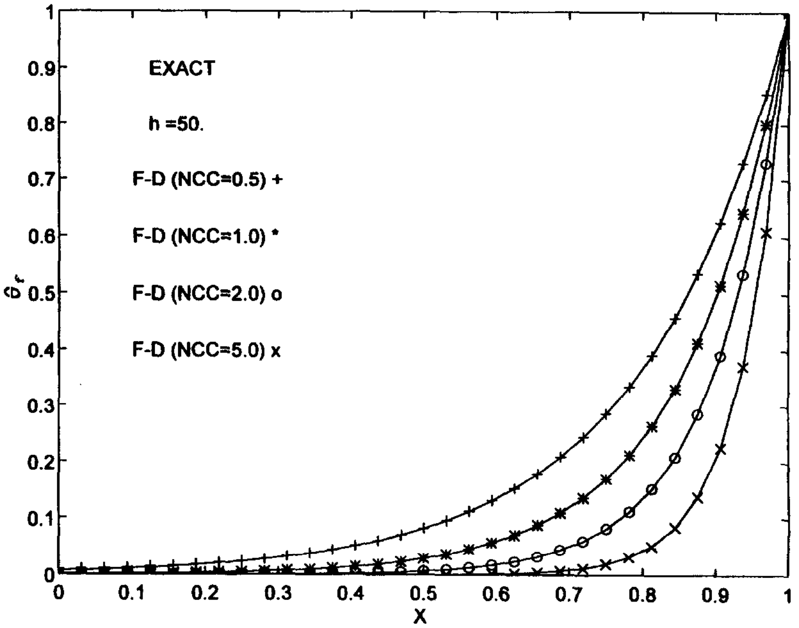

Fig. 3. Fin temperature distributions computed from FDE and exact solution for $h = 50.0$.

flow and temperature fields are obtained, and computed values of TT vs the elastic number E with the Prandtl number $Pr = 1.0$, 10.0, 100.0 are shown in Fig. 4. Variations indicated that TT is linear proportional to the elastic number E and values of TT are larger with a larger Prandtl number. Also, variations in TT vs the Prandtl number with the elastic number $E = 0.3$, 0.1, 0.001 are shown in Fig. 5. Figures 4 and 5 all indicate that a larger temperature gradient is present with a larger elastic number and the Prandtl number.

Figures 6 and 7, like Figs 4 and 5, illustrate the relationships among TT, Pr and E for the flat-plate flow field ($\beta = 0$) at the location $X = 0.1$. A linear relationship between TT and E is shown in Fig. 6. Both Figs 6 and 7 indicate that values of TT are larger with a medium value of Pr. Variations of TT along the flat plate with $Pr = 1.0$ and $E = 0.1$, 0.2, 0.3 are shown in Fig. 8. A stronger local effect is shown at locations near the fin tip. The present local similar solution approaches self-similar behavior at locations $X > 0.3$. Numerical values of $f_0''(0)$, $g_0'(\xi, 0)$, $f_1''(0)$,

Table 1. f''_0 and f''_1 vs β

β	$f''_0(0)$ Ref. [5]	Present solution	Errors	$f''_1(0)$ Ref. [5]	Present solution	Errors
0.05	0.5311	0.5312	0.0001	0.8214	0.8278	0.0064
0.10	0.5870	0.5871	0.0001	0.5296	0.5279	0.0017
0.20	0.6867	0.6869	0.0002	0.3009	0.2985	0.0024
0.30	0.7748	0.7751	0.0003	0.1418	0.1401	0.0017
0.40	0.8544	0.8548	0.0004	−0.0112	−0.0123	0.0011
0.50	0.9277	0.9282	0.0005	−0.1708	−0.1717	0.0035
0.60	0.9958	0.9965	0.0007	−0.3409	−0.3419	0.0010
0.80	1.1202	1.1211	0.0009	−0.7164	−0.7181	0.0017
1.00	1.2326	1.2337	0.0011	−1.1390	−1.1420	0.0030
1.20	1.3357	1.3371	0.0014	−1.6064	−1.6112	0.0048
1.60	1.5215	1.5234	0.0019	−2.6641	−2.6744	0.0003

Table 2. $TT0$ vs Pr ($\beta = 0.0$)

Pr	$TT0$ Ref. [28]	$TT0$ present	Errors
0.3	0.2148	0.2207	0.0059
0.6	0.2770	0.2777	0.0007
0.72	0.2955	0.2960	0.0005
1.0	0.3321	0.3323	0.0002
2.0	0.4223	0.4227	0.0004
3.0	0.4850	0.4856	0.0006
6.0	0.6133	0.6145	0.0012
10.0	0.7281	0.7304	0.0013
30.0	1.0517	1.0602	0.0085
60.0	1.3255	1.3459	0.0204
100.0	1.5718	1.6106	0.0388

$g'_1(\xi, 0)$, $\theta'(\xi, 0)$, $TT0$, $TT1$ and TT for stagnation flow field ($\beta = 1.0$) and flat-plate flow field ($\beta = 0.0$, $\xi = 2$) vs Pr and E are listed in Tables 3–6, respectively. These values vs locations X for flat-plate flow (from local similar solution) are listed in Table 7.

Variations in the local heat-transfer coefficient (h) along the fin with $N_{cc} = 0.5, 1.0, 1.5$ and 2.0 are shown in Figs 9–12, respectively. Computations of the local heat transfer coefficient are made by combining a stagnation flow at the fin tip and a flat-plate flow on side surfaces of the fin. Results show that the local heat transfer coefficient and heat transfer increase dramatically near the fin tip. It also points out that using a stagnation flow at the fin tip is much closer to the physical situations of the present problem. The value of the conjugate approach is clearly demonstrated by using a stagnation flow as an end condition. In addition, these figures indicate that higher E values can enhance the heat transfer performance (higher local heat transfer coefficient) no matter how the N_{cc} varies. However, variations of N_{cc} from 0.5 to 2.0 have insignificant effects on the local heat transfer coefficient.

The conjugate fin temperature distributions along the fin with $E = 0.001, 0.01$ and 0.1 are shown in Figs 13–15, respectively. The tip temperature is lower and

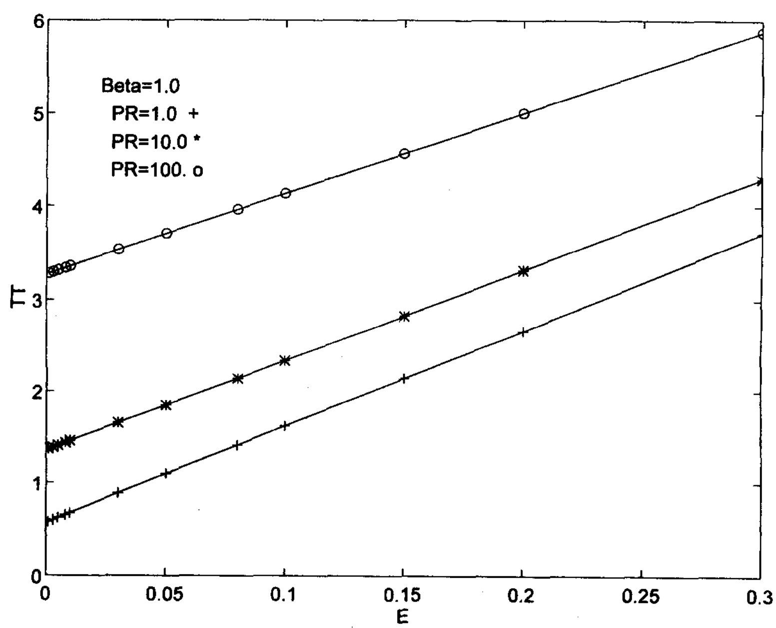

Fig. 4. TT vs E for $\beta = 1.0$, $Pr = 1.0, 10.0$ and 100.0.

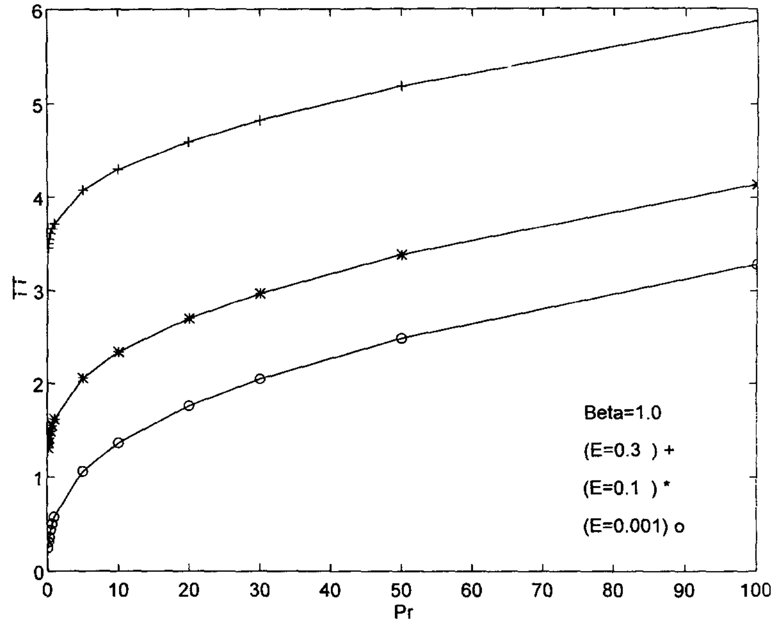

Fig. 5. TT vs Pr for $\beta = 1.0$, $E = 0.001$, 0.1 and 0.3.

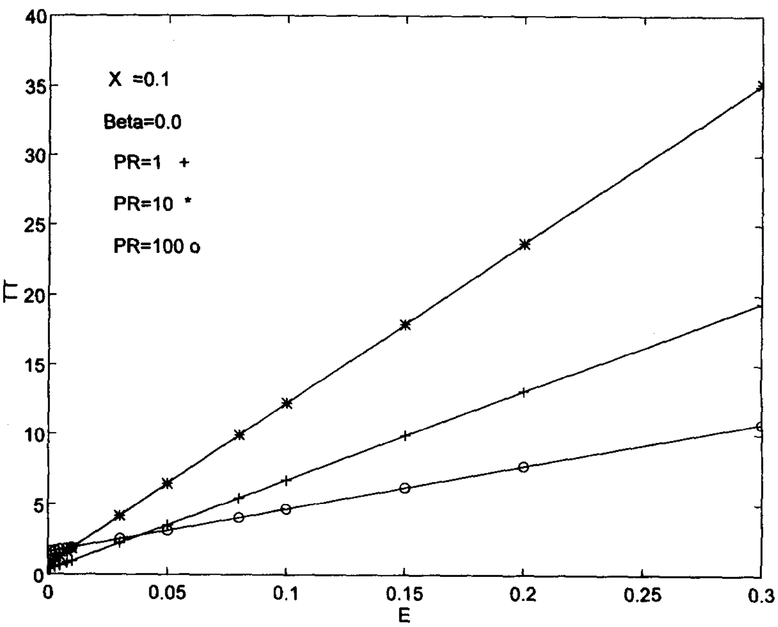

Fig. 6. TT vs E for $\beta = 0.0$, $X = 0.1$, $Pr = 1.0$, 10.0 and 100.0.

temperature gradients along the fin are higher with larger values of N_{cc} and E. These figures again indicate that the elastic nature of the fluid and a higher conductivity of the fin material can enhance the heat transfer performance of the fin.

4. CONCLUSION

In the present study, a second-grade viscoelastic fluid flow has been introduced into analyses of a conjugate heat transfer problem of conduction in a solid flat-plate fin and a forced convection in flow. The present conjugate problem is a hybrid system of the ordinary convective problem with a constant wall temperature. A local heat transfer coefficient is obtained from numerical solutions. Other features in the study included are a generalized Falkner–Skan flow derivation, and a stagnation flow field at fin tip and a flat-plate flow on fin surfaces.

Numerical results in the present study indicate that elastic effect E in the flow can increase the local heat transfer coefficient and enhance the heat transfer of a fin. Also, the same as the results from Newtonian fluid flow and conduction analysis of a fin, a better heat

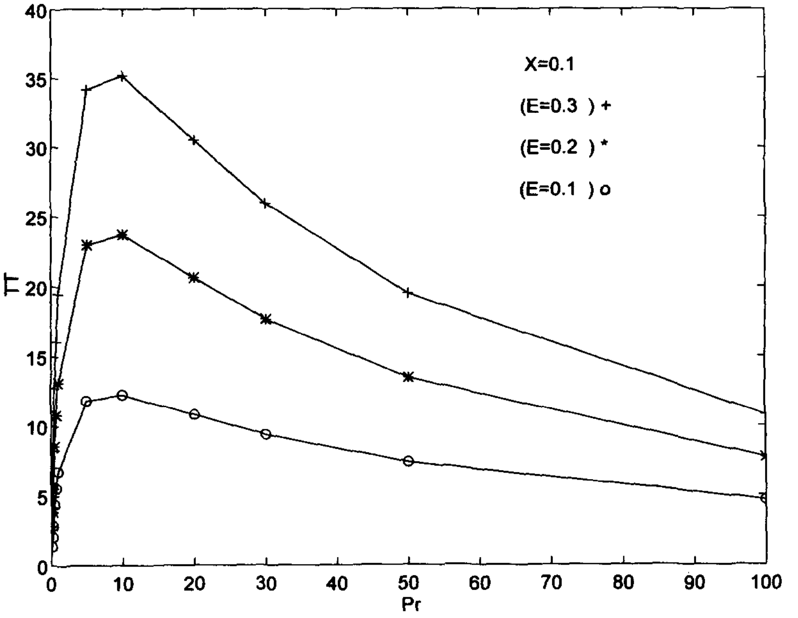

Fig. 7. TT vs Pr for $\beta = 0.0$, $X = 0.1$, $E = 0.1$, 0.2 and 0.3.

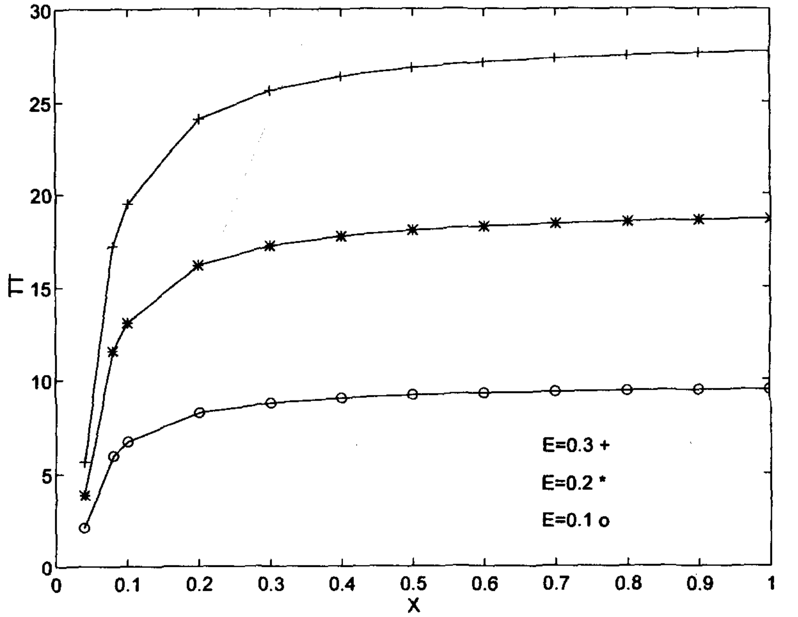

Fig. 8. TT vs X for $\beta = 0.0$, $Pr = 1.0$, $E = 0.1$, 0.2 and 0.3.

Table 3. TT, $TT0$ and $TT1$ vs Pr for stagnation flow field ($\beta = 1.0$, $E = 0.001$)

Pr	$f_0''(0)$	$g_0'(0)$	$TT0$	$f_1''(0)$	$g_1'(0)$	$TT1$	$\theta'(0)$	TT
0.1	1.2337	−0.2442	0.2442	−1.1420	−10.6862	10.6862	−0.2549	0.2549
0.2	1.2337	−0.3033	0.3033	−1.1420	−10.6584	10.6584	−0.3139	0.3139
0.3	1.2337	−0.3538	0.3538	−1.1420	−10.6312	10.6312	−0.3645	0.3645
0.5	1.2337	−0.4344	0.4344	−1.1420	−10.5802	10.5802	−0.4450	0.4450
−0.7	1.2337	−0.4972	0.4972	−1.1420	−10.5346	10.5346	−0.5077	0.5077
1.0	1.2337	−0.5723	0.5723	−1.1420	−10.4759	10.4759	−0.5828	0.5828
5.0	1.2337	−1.0524	1.0524	−1.1420	−10.0690	10.0690	−1.0625	1.0625
10.0	1.2337	−1.3578	1.3578	−1.1420	−9.8073	9.8073	−1.3677	1.3677
20.0	1.2337	−1.7521	1.7521	−1.1420	−9.4972	9.4972	−1.7616	1.7616
30.0	1.2337	−2.0378	2.0378	−1.1420	−9.2988	9.2988	−2.0471	2.0471
50.0	1.2337	−2.4759	2.4759	−1.1420	−9.0380	9.0380	−2.4850	2.4850
100.0	1.2337	−3.2688	3.2688	−1.1420	−8.6760	8.6760	−3.2775	3.2775

Table 4. TT, $TT0$ and $TT1$ vs Pr flat-plate flow field ($\beta = 0$, $E = 0.001$, $X = 0.5$, $\xi = 2$)

Pr	$f_0''(0)$	$g_0'(0)$	$TT0$	$f_1''(0)$	$g_1'(0)$	$TT1$	$\theta'(0)$	TT
0.1	0.4696	−0.2424	0.1714	5.1602	−18.5369	13.1075	−0.2610	0.1845
0.2	0.4696	−0.2789	0.1972	5.1602	−30.2570	21.3949	−0.3092	0.2186
0.3	0.4696	−0.3121	0.2207	5.1602	−45.2937	32.0275	−0.3574	0.2527
0.5	0.4696	−0.3686	0.2607	5.1602	−74.8492	52.9264	−0.4435	0.3136
0.7	0.4696	−0.4145	0.2931	5.1602	−98.3691	69.5574	−0.5129	0.3627
1.0	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−0.5949	0.4207
5.0	0.4696	−0.8170	0.5777	5.1602	−270.1608	191.0325	−1.0871	0.7687
10.0	0.4696	−1.0329	0.7304	5.1602	−312.8194	221.1967	−1.3457	0.9516
20.0	0.4696	−1.3061	0.9235	5.1602	−320.7817	226.8269	−1.6269	1.1504
30.0	0.4696	−1.4993	1.0602	5.1602	−312.1334	220.7116	−1.8115	1.2809
50.0	0.4696	−1.7868	1.2634	5.1602	−292.7731	207.0219	−2.0795	1.4705
100.0	0.4696	−2.2778	1.6106	5.1602	−260.2552	184.0282	−2.5381	1.7947

Table 5. θ', $TT0$ and $TT1$ vs E for stagnation flow field ($\beta = 1.0$, $Pr = 1.0$)

E	$f_0''(0)$	$g_0'(0)$	$TT0$	$f_1''(0)$	$g_1'(0)$	$TT1$	$\theta'(0)$	TT
0.001	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−0.5828	0.5828
0.003	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−0.6037	0.6037
0.005	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−0.6247	0.6247
0.008	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−0.6561	0.6561
0.010	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−0.6770	0.6770
0.030	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−0.8866	0.8866
0.050	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−1.0961	1.0961
0.080	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−1.4104	1.4104
0.100	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−1.6199	1.6199
0.150	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−2.1437	2.1437
0.200	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−2.6675	2.6675
0.300	1.2337	−0.5723	0.5723	−1.142	−10.4759	10.4759	−3.7151	3.7151

Table 6. Field TT, $TT0$ and $TT1$ vs E flat-plate flow ($\beta = 0.0$, $Pr = 1.0$, $X = 0.5$, $\xi = 2$)

E	$f_0''(0)$	$g_0'(0)$	$TT0$	$f_1''(0)$	$g_1'(0)$	$TT1$	$\theta'(0)$	TT
0.001	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−0.5949	0.4207
0.003	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−0.8449	0.5974
0.005	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−1.0948	0.7741
0.008	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−1.4697	1.0393
0.010	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−1.7197	1.2160
0.030	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−4.2191	2.9834
0.050	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−6.7186	4.7508
0.080	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−10.4678	7.4019
0.100	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−12.9673	9.1692
0.150	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−19.2159	13.5877
0.200	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−25.4646	18.0062
0.300	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−37.9619	26.8431

Table 7. Flat-plate flow field TT, $TT0$ and $TT1$ vs X at the wall ($\beta = 0.0$, $Pr = 1.0$, $E = 0.001$)

X	$f_0''(0)$	$g_0'(0)$	$TT0(0)$	$f_1''(0)$	$g_1'(0)$	$TT1(0)$	$\theta'(0)$	$TT(0)$
0.04	0.4696	−0.4699	0.3323	5.1602	−24.9832	17.6658	−0.4949	0.3500
0.08	0.4696	−0.4699	0.3323	5.1602	−79.3257	56.0917	−0.5493	0.3884
0.10	0.4696	−0.4699	0.3323	5.1602	−90.1942	63.7769	−0.5601	0.3961
0.20	0.4696	−0.4699	0.3323	5.1602	−111.9311	79.1473	−0.5819	0.4114
0.30	0.4696	−0.4699	0.3323	5.1602	−119.1768	84.2707	−0.5891	0.4166
0.40	0.4696	−0.4699	0.3323	5.1602	−122.7996	86.8324	−0.5927	0.4191
0.50	0.4696	−0.4699	0.3323	5.1602	−124.9733	88.3695	−0.5949	0.4207
0.60	0.4696	−0.4699	0.3323	5.1602	−126.4225	89.3942	−0.5964	0.4217
0.70	0.4696	−0.4699	0.3323	5.1602	−127.4575	90.1261	−0.5974	0.4224
0.80	0.4696	−0.4699	0.3323	5.1602	−128.2339	90.6750	−0.5982	0.4230
0.90	0.4696	−0.4699	0.3323	5.1602	−128.8377	91.1020	−0.5988	0.4234
1.00	0.4696	−0.4699	0.3323	5.1602	−129.3207	91.4436	−0.5993	0.4237

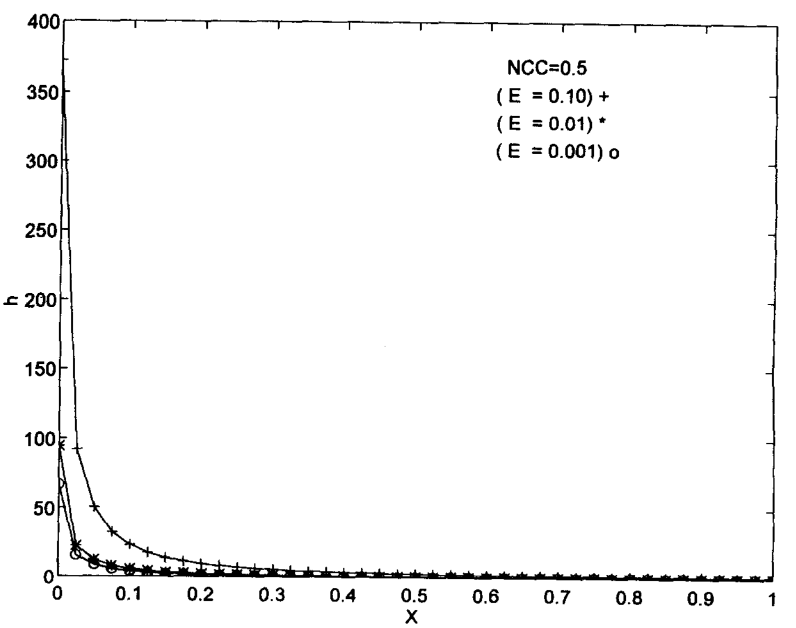

Fig. 9. Local convective heat transfer coefficient distributions for $N_{cc} = 0.5$, $\beta = 0.0$, and $E = 0.001$, 0.01 and 0.1.

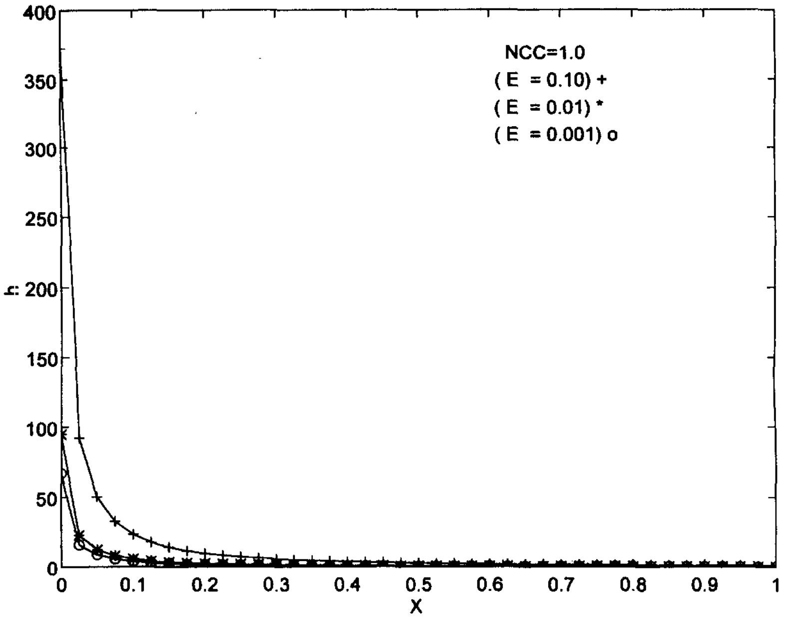

Fig. 10. Local convective heat transfer coefficient distributions for $N_{cc} = 1.0$, $\beta = 0.0$, and $E = 0.001$, 0.01 and 0.1.

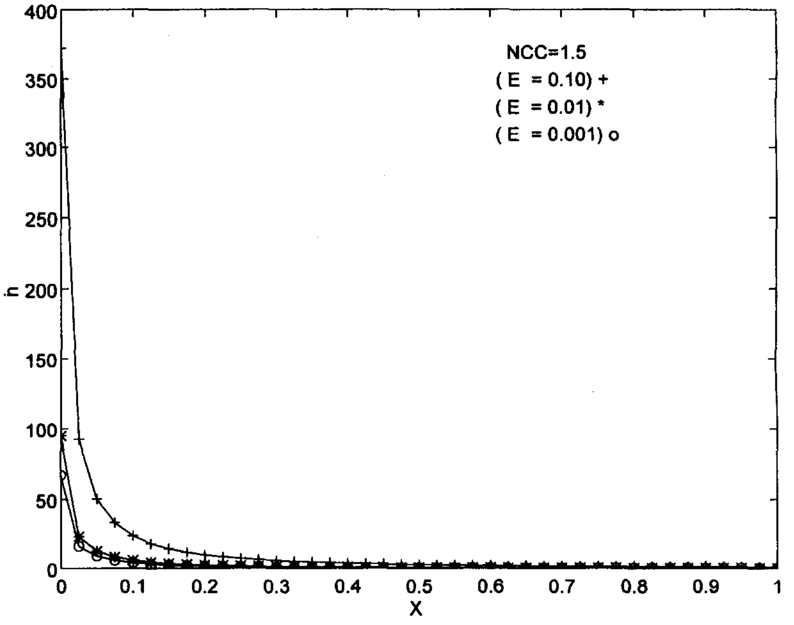

Fig. 11. Local convective heat transfer coefficient distributions for $N_{cc} = 1.5$, $\beta = 0.0$, and $E = 0.001$, 0.01 and 0.1.

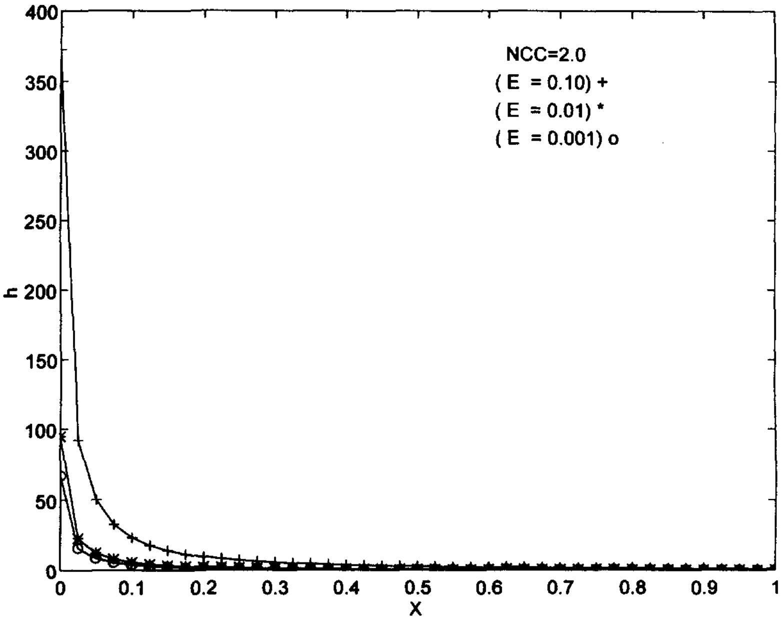

Fig. 12. Local convective heat transfer coefficient distributions for $N_{cc} = 2.0$, $\beta = 0.0$, and $E = 0.001$, 0.01 and 0.1.

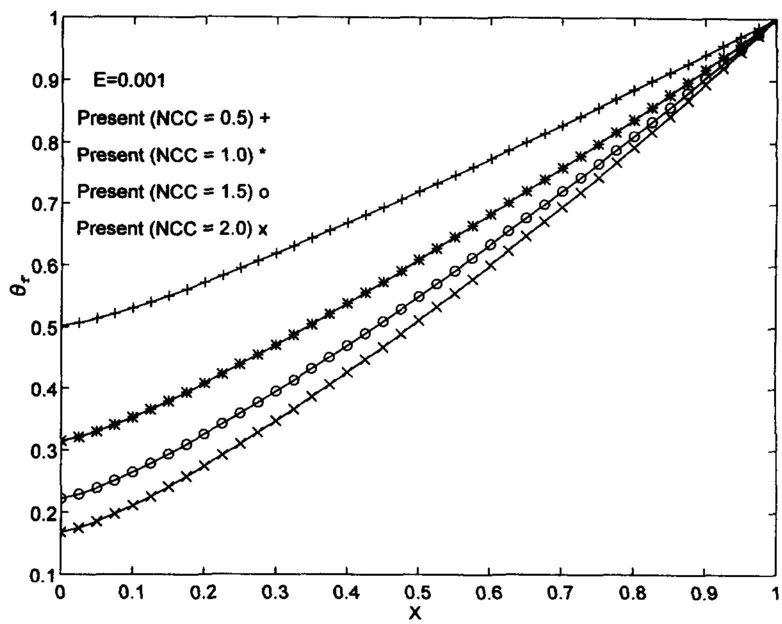

Fig. 13. Conjugate fin temperature distributions for $E = 0.001$, $N_{cc} = 0.5$, 1.0, 1.5 and 2.0.

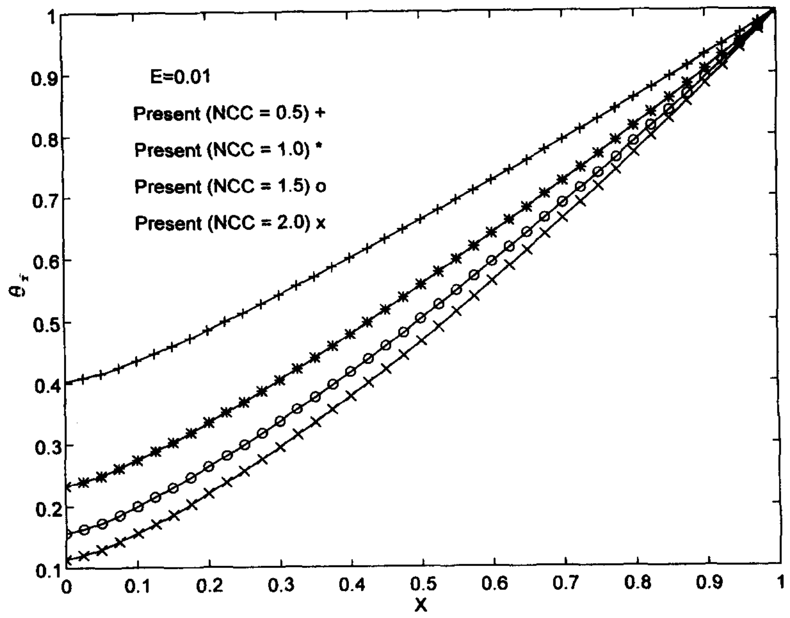

Fig. 14. Conjugate fin temperature distributions for $E = 0.01$, $N_{cc} = 0.5$, 1.0, 1.5 and 2.0.

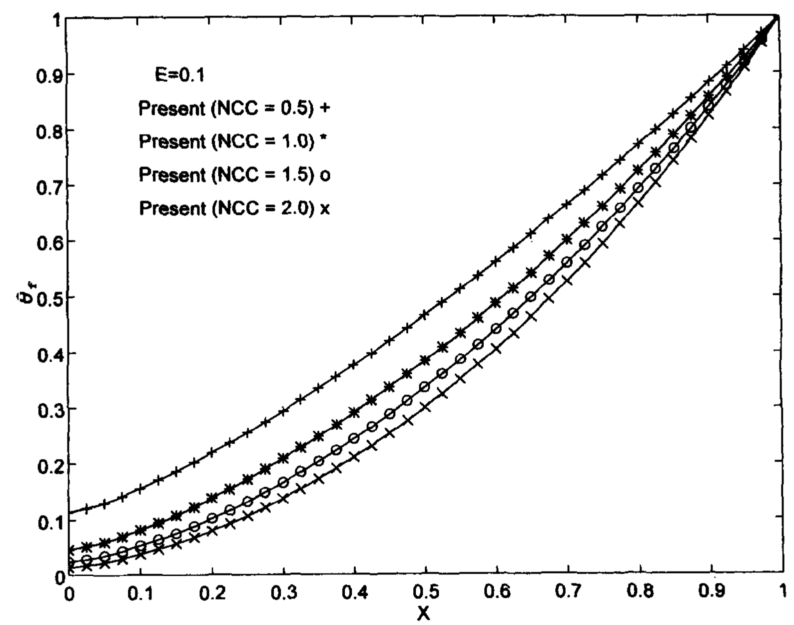

Fig. 15. Conjugate fin temperature distributions for $E = 0.1$, $N_{cc} = 0.5$, 1.0, 1.5 and 2.0.

transfer is obtained with a larger N_{cc} and a larger *Pr*. With some modifications, the present method can be applied to the conjugate problems with other boundary-layer flow fields.

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